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Short Communication

Fundamental frequencies of annular plates with movable edges

C.Y. Wang*

Department of Mathematics, Michigan State University, East Lansing, MI 48824, USA

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Abstract

Vibrations of thin annular plates with at least one sliding or movable edge are considered. The fundamental frequencies for the seven cases may correspond to axisymmetric or non-axisymmetric modes.

1. Introduction

There exists extensive literature on the vibration of plates, especially with clamped, simply supported, or free edges (e.g. Refs. [1,2]). Recently, Wang and Wang [3] pointed out that, for some annular plates with small cores, the fundamental frequency is greatly lowered due to a mode change from axisymmetric to a non-axisymmetric one.

Very few sources considered the fourth basic boundary condition, that of the movable or sliding edges. On these boundaries the edges do not rotate but are free to move laterally. Plates with movable edges model moving parts such as piston heads. McLeod and Bishop [4] formulated the governing equations, but only solved for the full circular plate frequencies. In this note we shall present the results for annular plates.

*Fax: +517 432 1562.
E-mail address: cywang@mth.msu.edu.

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2. Formulation

The general form of the lateral displacement of the vibration of a classical thin plate can be expressed as $u(r)\cos(n\theta)e^{i\omega t}$, where (r, θ) are polar coordinates, n is an integer and ω is the frequency. The function $u(r)$ is a linear combination of the Bessel functions $J_n(kr)$, $Y_n(kr)$, $I_n(kr)$, $K_n(kr)$, where $k = (\text{radius})[(\text{density})(\omega)^2/(\text{flexural rigidity})]^{1/4}$ is the square root of the non-dimensional frequency [1]. The normalized bending moment is

$$M(r) = u''(r) + v\left[\frac{1}{r}u'(r) - \frac{n^2}{r^2}u(r)\right]. \tag{1}$$

The normalized effective shear force is

$$V(r) = u'''(r) + \frac{1}{r}u''(r) - [1 + n^2(2 - v)]\frac{1}{r}u'(r) + n^2(3 - v)u(r). \tag{2}$$

Here v is Poisson's ratio. For a full plate with a movable edge, the boundary conditions are that the displacement and moment are bounded at the center, and that on the outer edge

$$u'(1) = 0, \quad V(1) = 0. \tag{3}$$

Using only the bounded functions J_n and I_n, Eqs. (3) yield an exact characteristic determinant for the eigenvalue k. A root finding scheme gives Table 1 for $v = 0.3$. For $n = 0$, the frequencies are independent of Poisson's ratio. McLeod and Bishop [4] gave less accurate values of 3.84, 7.02, 10.18 for s (number of nodal circles) equal to 1, 2, 3. Note that the case $s = n = 0$ is not included since the plate can move vertically as a rigid body. Consulting Table 1, the fundamental frequency is 1.7557 corresponding to a non-axisymmetric mode $s = 0$ and $n = 1$.

The results of Ref. [4] for $n > 0$ ($v = 0.33$) are also somewhat off. For $n = 1$ we obtain 1.7596, 5.3292, 8.5358, 11.706 for $s = 0, 1, 2, 3$ while the values in Ref. [4] are 0, 5.33, 8.54, 11.7, respectively.

3. Annular plates

Consider an annular plate with (normalized) inner radius of b and outer radius of 1. There are seven kinds of combinations involving a movable edge. Let C, S, F, M represent clamped, simply

Table 1
Frequency k for a full plate with moving edge

s	n			
	0	1	2	3
0	—	1.7557	2.9639	4.1112
1	3.8317	5.3291	6.7010	8.0081
2	7.0156	8.5357	9.9680	11.344
3	10.174	11.706	13.170	14.585

s is the number of nodal circles and n is the number of nodal diameters ($v = 0.3$).

supported, free, and movable edge. Let the first letter denote the inner edge and the second letter the outer edge. The seven cases are CM, MC, SM, MS, MM, FM, and MF. None of these cases has been studied before. We shall be concentrating on the fundamental (lowest) frequency for $v = 0.3$.

For the CM case, the boundary conditions are

$$u(b) = 0, \quad u'(b) = 0, \tag{4}$$

$$V(1) = 0, \quad u'(1) = 0. \tag{5}$$

Now when $b \to 0$ the clamped core is equivalent to a nailed center. Such constraint increases the frequency for the $n = 0$ axisymmetric mode but not the $n > 0$ modes with nodal diameters. Thus the fundamental frequency limit is 1.7557 ($n = 1$) as $b \to 0$. On the other hand, when b is close to one, the plate resembles a long strip of width $1-b$ and one edge clamped, one edge movable. The characteristic equation is given by

$$\sin x \cosh x + \cos x \sinh x = 0, \quad x = k(1-b). \tag{6}$$

The lowest root is $x = 2.365$, or the fundamental frequency is asymptotically

$$k = \frac{2.365}{1-b} \tag{7}$$

corresponding to the axisymmetric $n = 0$ mode. Thus the vibration switches form the $n = 1$ mode to the $n = 0$ mode as the core radius is increased. The location of switch is found to be at $b = 0.118$ when $k = 2.471$. Such switching phenomenon is similar to that reported in Ref. [3].

For the MC case, the boundary conditions are the same as Eqs. (4) and (5) except b and 1 are interchanged. We find the axisymmetric $n = 0$ case gives the fundamental frequency for the whole range of b. When b is zero the plate becomes a full plate with clamped outer edge. The frequency is the root of

$$I_1(k)J_0(k) + I_0(k)J_1(k) = 0, \tag{8}$$

yielding $k = 3.1962$. For b close to one Eq. (7) describes the asymptotic behavior.

In the SM case the boundary conditions are

$$u(b) = 0, \quad M(b) = 0 \tag{9}$$

and Eqs. (5). Similar arguments as the CM case show the fundamental frequency is 1.7557 for $n = 1$ as $b \to 0$. For b close to one, a strip of $1-b$ width, one edge movable and one edge simply supported, gives the asymptotic fundamental frequency

$$k = \frac{\pi}{2(1-b)} \tag{10}$$

corresponding to the $n = 0$ mode. The switching occurs at $b = 0.103$ when $k = 2.148$.

The MS case gives axisymmetric vibrations for the fundamental frequency. Eq. (10) still governs the behavior for b close to one. When the core is small, the vibration mimics that of a full simply supported plate. The frequency is governed by

$$[2vI_1(k) + kI_2(k)]J_0(k) + [2kJ_0(k) + 2vJ_1(k) - kJ_2(k)]I_0(k) = 0. \tag{11}$$

For $v = 0.3$, the first root is $k = 2.2215$.

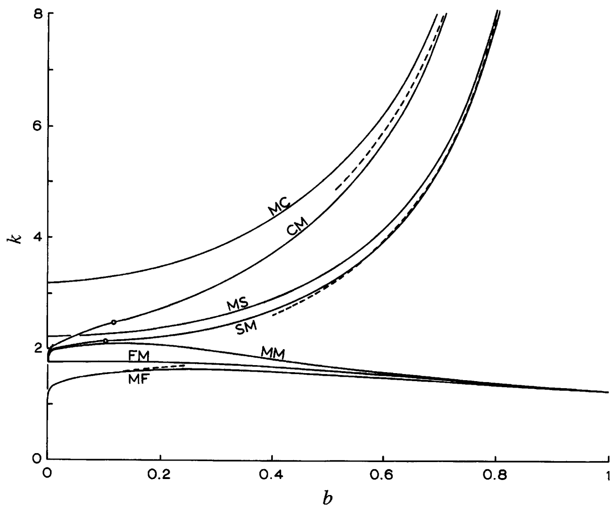

Fig. 1. The fundamental frequency of an annular plate with movable edge condition. Dashed lines are asymptotic approximations, Eqs. (7,10,13). Small circles represent locations of mode changes.

The MM case utilizes Eq. (5) both at b and at 1. We find the lowest frequency always correspond to the $n = 1$ mode. Unlike the previous cases, the frequency does not rise monotonically with b. The fundamental frequency is 1.7557 when $b = 0$, then rises to a maximum of 2.107 when $b = 0.131$, and decreases to 1.245 when b is close to one.

The boundary conditions for the FM case are Eq. (5) and

$$V(b) = 0, \quad M(b) = 0. \tag{12}$$

The $n = 1$ mode dominates the fundamental frequency which starts from 1.7557 at $b = 0$ and decreases to 1.233 when $b \to 1$.

The MF case is also governed by the $n = 1$ mode. However it starts singularly from zero. Using asymptotic expansions similar to Ref. [3] we find for small b,

$$k \sim \frac{2}{\{|\ln b| + (1 + 22\nu + 9\nu^2)/[8(3 - 2\nu - \nu^2)]\}^{1/4}}. \tag{13}$$

A maximum of 1.634 is reached at $b = 0.263$. Then the fundamental frequency decreases to 1.233 when $b \to 1$.

Fig. 1 shows the results of all seven cases.

4. Conclusions

This note complements the published results on annular plates by considering the seven cases which involve the movable edge condition. As the core size is increased, we find the fundamental frequency my increase or decrease. The fundamental mode may be axisymmetric, non-axisymmetric, or a mixture of both. Our graphical Fig. 1 (instead of numerous tables) illuminates the behavior of the fundamental frequency.

References

[1] A.W. Leissa, *Vibration of Plates*, NASA SP-160, 1969.
[2] G.N. Weisensel, Natural frequency information for circular and annular plates, *Journal of Sound and Vibration* 133 (1989) 129–134.
[3] C.Y. Wang, C.M. Wang, Examination of the fundamental frequencies of annular plates with small core, *Journal of Sound and Vibration* 280 (2005) 1116–1124.
[4] A.J. McLeod, R.E.D. Bishop, The forced vibration of circular flat plates, *Mechanical Engineering Science Monograph* 1 (1965) 1–33.